the original circle must be expanded by reducing $|a_2|$. In case of a very high insertion loss, some difficulties in recording the standing wave pattern may be encountered because the power level in the slotted waveguide drops when one tries to expand the circle too much. However, it is still possible to measure an insertion loss of some 40 dB, and this with a better accuracy than with the former methods.

A theoretical study of the experimental error has been performed, but is too elaborate to be given in any detail. The theory shows that accuracies of one percent on the magnitude and 2° on the phase angle are easily achieved with run of the mill microwave equipment.

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[2] J. E. Pippin, "Scattering matrix measurements on nonreciprocal microwave devices," *Proc. IRE*, vol. 44, p. 110, January 1956.
[3] H. M. Altschuler, "Nonreciprocal two-part measurement based on an averaging technique," *Proc. IRE*, vol. 45, p. 1293, September 1957.

Comments on "A Technique for Measuring Individual Modes Propagating in Overmoded Waveguide"

In a recent paper, Levinson and Rubinstein[1] presented a technique for measuring individual modes propagating in overmoded waveguide. Although the measurement technique described therein is practical and effective, one of the conclusions drawn from the results appears to be misleading. The measurement consists of sampling the voltage amplitude and relative phase of a propagating multimode 7 GHz signal by means of a series of probes located about the walls of a section of WR 650 waveguide. From this voltage data and the appropriate calculations, the relative power propagating in each mode through the WR 650 waveguide is determined. One advantage of measuring in oversized waveguide is that the modes of interest are far from cutoff for the tests performed at 7 GHz. According to Taub[2] a maximum error of only six percent can occur in the calculations of relative power if, for simplicity, free-space impedance is assumed instead of the individual wave impedance for each mode. As the paper correctly indicates, the relative power ratios determined for the modes propagating in the oversized (WR 650) waveguide are the same as the relative power ratios of the modes propagating in the standard (WR 284) waveguide, assuming very little mode conversion caused by the connecting tapered section.

The misleading conclusion implied in the paper (Fig. 10 and associated text), however, is that the relative *amplitude* ratios for the modes propagating in the oversized waveguide are the same as the amplitude ratios for the respective modes propagating in the standard waveguide. This conclusion is not correct because the ratio of the mode wave impedances are different in the two different waveguide sizes. Hence, as two modes having a constant power ratio propagate from a waveguide cross section in which one mode is near cutoff to a larger waveguide cross section in which both modes are far from cutoff, the mode amplitude ratio decreases, assuming no mode conversion, according to the square root of the appropriate wave impedances. Applying this correction will result in good agreement between the reported tests and the results of the theoretical analysis by Felson.[3]

Finally, the paper does not indicate whether the relative phase data in Figs. 6, 7, 8, and 10 is referenced at the test probes or at the particular mode-exciting device. Because of the dispersive nature of the taper and the standard size waveguide, the relative mode phases at these locations are significantly different.

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[1] D. S. Levinson and I. Rubinstein, "A technique for measuring individual modes propagating in overmoded waveguide," *IEEE Trans. on Microwave Theory and Techniques* vol. MTT-14, pp. 310–322, July 1966.

[2] J. J. Taub, "A new technique for multimode power measurement," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 496–505, November 1962.

Author's Reply[4]

We would like to thank Puttre for his comments on our paper on mode measurements[1] and offer the following clarifications. Puttre points out that it is implied that the relative mode amplitude ratios measured in the oversize guide are the same in the standard waveguide. Also, he has raised the question as to the plane at which the reported relative phase information was referenced, and noted that any translation of this data to the mode-excited device must account for the dispersive properties of the taper and standard waveguide. Concerning this last question, all phase data given are referenced at the plane of the measurement probes. Also, since at the time this work was performed our prime interest was in specifying the over-moded aperture illumination at the plane of the large end of the taper, any translation of the measured relative phase was carried back only to that plane. Even here, although the corrections were small, we did account for the relative phase dispersion between modes to obtain the best accuracy.

With reference to the translation of the relative mode amplitudes from the oversize to standard waveguide, here again we were concerned only with specifying their measure at the radiating aperture. However, sufficient information is available from the measurements whereby the mode amplitude ratios measured in one size guide can be readily translated to the ratio in any other size guide. We feel that Puttre's point is well taken, and, for the sake of completeness, it will be shown here how the translations are accomplished. It should first be pointed out that the ratio of the amplitudes of any TE_{mn} modes does not change where the waveguides being considered have the same aspect ratio, i.e., $a_1/b_1 = a_2/b_2$. This is due to the fact that the manner in which the measurements are performed yields relative amplitudes which are proportional to the amplitudes of the electric field components only. In general, however, the amplitude ratios of modes in different size waveguides are different, and can be related through the respective cutoff frequencies for the modes in each size guide, as is shown below.

Using the notation as in Moreno,[5] and considering, for example, the translation of the ratio of two TE_{mn} modes, we get

$$\frac{A_{TE_{mn1}}}{A_{TE_{mn2}}} \propto \frac{B_{mn1}K^2_{mn1}}{B_{mn2}K^2_{mn2}} \qquad (1)$$

where the term on the left side is the measured amplitude ratio, and

$$K^2 = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) = \left(\frac{2\pi}{c}\right)^2 f_c^2. \qquad (2)$$

Letting the bracket subscript denote waveguide sizes 1 and 2, the translation ratio is given by

$$\frac{[A_{mn1}/A_{mn2}]_1}{[A_{mn1}/A_{mn2}]_2} = \frac{[f_{cmn1}/f_{cmn2}]_1^2}{[f_{cmn1}/f_{cmn2}]_2^2}. \qquad (3)$$

As noted before, for waveguides having the same aspect ratios, the ratio of the cutoff frequencies remains the same, and thus the mode amplitude ratios are the same.

In performing the above calculation, the accuracy is not dependent on how near to cutoff the respective modes are. For TE_{mn} modes, this dependency shows up only in the expressions for the magnetic field components. The converse is true when considering TM_{mn} modes, and in the following equations for translating the electric field amplitude ratios of TM_{mn} modes, the computations could blow-up if one of the modes is near cutoff in one of the guides.

For TM_{mn} modes, then, we have in general,

$$\frac{A_{TM_{mn1}}}{A_{TM_{mn2}}} \propto \frac{A_{mn1}\beta_{mn1}K^2_{mn2}}{A_{mn2}\beta_{mn2}K^2_{mn1}} \qquad (4)$$

where

$$\beta^2 = \frac{\omega^2}{c^2} - K^2 = \left(\frac{2\pi}{c}\right)^2 (f^2 - f_c^2). \qquad (5)$$

The translation ratio for the two different size guides is then,

$$\frac{[A_{mn1}/A_{mn2}]_1}{[A_{mn1}/A_{mn2}]_2} = \frac{[f_{cmn2}/f_{cmn1}]_1^2}{[f_{cmn2}/f_{cmn1}]_2^2} \cdot \left\{\frac{[f^2 - f^2_{cmn1}/f^2 - f^2_{cmn2}]_1}{[f^2 - f^2_{cmn1}/f^2 - f^2_{cmn2}]_2}\right\}^{1/2}. \qquad (6)$$

For a combination of TE_{mn} and TM_{mn} modes, the translation ratio is,

[3] L. B. Felson and N. Marcuvitz, "Modal analysis and synthesis of electromagnetic fields," Microwave Research Inst., Polytechnic Inst. of Brooklyn, N. Y., Rept. R-446-55 (a) and (b), February 1956.

[4] Manuscript received August 23, 1966.

[5] T. Moreno, *Microwave Transmission Design Data*. New York: Dover, 1958, p. 115.

$$\frac{[A_{\text{TE}_{mn}}/A_{\text{TM}_{mn}}]_1}{[A_{\text{TE}_{mn}}/A_{\text{TM}_{mn}}]_2} = \frac{[f_{c\text{TM}}/f_{c\text{TE}}]_1^2}{[f_{c\text{TM}}/f_{c\text{TE}}]_2^2} \frac{[f^2 - f^2_{c\text{TM}}]_2}{[f^2 - f^2_{c\text{TM}}]_1}. \quad (7)$$

Thus it has been shown that the relative mode amplitudes in the standard waveguide can be readily obtained once the relative amplitudes in the oversize waveguide have been determined.

In addition to the foregoing, we wish to note a correction and an omission. It has been pointed out by M. Sirel of Laboratorie Central Des Ponts Et Chaussees, that reference [11] should have referred to *Electronic and Radio Engineer*, not *Electronics*. Also, due to an oversight, acknowledgment was not made to J. F. Ramsey and Dr. P. A. McInnes, of AIL, for providing the theoretical and measured multimode antenna data used in this effort.

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Constructive Coupling in Directional Couplers

During the course of some diagnostic studies[1] of varactor harmonic generators, we discovered a surprising result using the experimental setup shown in Fig. 1. A signal source is connected through an isolator to port 1 of a 10-dB directional coupler, with the output load connected to port 4 through a slide-screw tuner. Port 2 is internally terminated, and port 3 has connected to it a slide-screw tuner and termination.

By simultaneously adjusting the two slide-screw tuners it was possible to maximize the power delivered to the output load. The reader is invited to pause at this point and estimate the amount of this maximum power for a 10-dB directional coupler. Would you believe a net insertion loss from the generator to the load of 10 dB, 7 dB, 3 dB? The answer, surprisingly, is approximately 3 dB.

This result can be proved using the scattering matrix formalism. For the circuit of Fig. 1 the following matrix equation holds

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & A & 0 & jB \\ A & 0 & jB & 0 \\ 0 & jB & 0 & A \\ jB & 0 & A & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ \Gamma_3 b_3 \\ \Gamma_4 b_4 \end{bmatrix} \quad (1)$$

where a_k and b_k are the incident and reflected waves on port k, and Γ_3 and Γ_4 are the voltage reflection coefficients of the tuner and load combinations at ports 3 and 4, respectively. B and A are taken to be real by proper

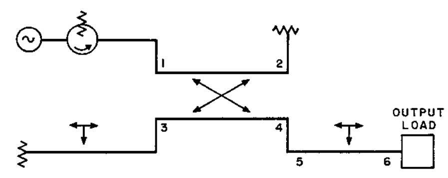

Fig. 1. Experimental setup using a directional coupler.

choice of reference planes. From (1) we have

$$b_3 = A\Gamma_4 b_4 \quad (2)$$

$$b_4 = jBa_1 + A\Gamma_3 b_3. \quad (3)$$

Substituting (2) into (3) gives

$$\frac{b_4}{a_1} = \frac{jB}{1 - A^2\Gamma_3\Gamma_4}. \quad (4)$$

Letting $\Gamma_3 = |\Gamma_3| e^{j\rho_3}$ and $\Gamma_4 = |\Gamma_4| e^{j\rho_4}$ in (4) and noting that $B^2 = 1 - A^2$ gives

$$\frac{|b_4|^2}{|a_1|^2} = \frac{1 - A^2}{1 + A^4|\Gamma_3|^2|\Gamma_4|^2 - 2A^2|\Gamma_3|\,|\Gamma_4| \cos(\rho_3 + \rho_4)}. \quad (5)$$

Since the tuner is lossless, the power absorbed in the output load is $|b_4|^2(1 - |\Gamma_4|^2)$, and the ratio of load power to generator power is

$$\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{1 + A^4|\Gamma_3|^2|\Gamma_4|^2 - 2A^2|\Gamma_3|\,|\Gamma_4| \cos(\rho_3 + \rho_4)}. \quad (6)$$

Since $A^2|\Gamma_3|\,|\Gamma_4|$ is positive and less than, or equal to, unity, this is a maximum for

$$\cos(\rho_3 + \rho_4) = 1$$

$$|\Gamma_3| = 1.$$

With these values, (6) becomes

$$\left.\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2}\right|_{\max} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{(1 - A^2|\Gamma_4|)^2}. \quad (7)$$

When this expression is further maximized with respect to $|\Gamma_4|$, one obtains

$$|\Gamma_4|_{\text{opt}} = A^2 \quad (8)$$

so that,

$$\left.\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2}\right|_{\max} = \frac{1}{1 + A^2}. \quad (9)$$

As the coupling diminishes, $A^2 \to 1$ and the net insertion loss approaches 3 dB. Calculations for 3 dB, 10 dB, and 20 dB directional couplers give net minimum insertion losses of 1.76 dB, 2.79 dB, and 2.99 dB, respectively. Of course, circuit losses will increase these values somewhat, especially for the 20-dB coupler.

The analysis presented above does not lend much physical insight into the mechanism responsible for the unexpected results. A physical explanation is offered as follows. Consider the reciprocal situation where the source and load are interchanged. The two slide-screw tuner probes are inserted deeply so as to form a resonant cavity between them at the operating frequency. For a weakly coupled directional coupler, this resonant cavity has nearly equal waves traveling in both directions. Thus, half the power goes to port 1 and half to port 2, corresponding to a net 3 dB of insertion loss from port 6 to port 1. Application of reciprocity proves the desired result.

Thus we see that the unusual result occurs because of a resonance associated with the in-line arms of a directional coupler. This example shows that one must avoid such resonances in measurement schemes in order to maintain accuracy.

Such a resonance characterizes the behavior of the resonant ring circuit.[2] In the resonant ring circuit, neglecting losses, all of the power ends up in the termination on port 2. In the circuit of Fig. 1, as $A^2 \to 1$ (weak coupling) half of the generator power ends up in the output load connected to port 4, one-fourth of the power is delivered to the port 2 termination, and one-fourth of the power is reflected from port 1.

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[1] These studies concerned spurious oscillations resulting when the input circuit presents favorable impedances at two frequencies whose sum is the input frequency.

[2] H. Golde, "Theory and measurement of Q in resonant ring circuits," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-8, pp. 560–564, September 1960.

Precision Design of Direct Coupled Filters

The direct coupled filter to be discussed in this correspondence consists of a length of transmission line with reflecting obstacles spaced at approximately half wavelength intervals. High-power capabilities can be achieved because the filter can be made without changing the size of the transmission line and, as will be described, without any special tuning devices. The following analysis is applicable for waveguides with inductive obstacles and also to coaxial lines with inductive posts. By duality it is also valid for coaxial or strip line with capacitive gaps.

For obstacles in rectangular waveguide, inductive posts were preferred to irises. The use of a programmed tape controlled milling machine is to be recommended for production runs of this class of filter. Each obstacle used is a pair of posts whose spacing along the guide and from each other can be carefully

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